

Optimal reactive current compensation for smart grids using linear programming: A novel algorithm with theoretical and real-world data validation

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ABSTRACT

This paper presents an innovative optimization approach for reactive current compensation in modern distribution networks, based on a novel algorithmic solution using linear programming techniques. The proposed method determines optimal shunt compensator parameters by effectively linearizing nonlinear systems in high-harmonic environments without requiring negative reactive elements. Unlike traditional methods, this approach ensures reliable compensator values across diverse operational scenarios, making it particularly valuable for smart grid applications where power quality and energy efficiency are crucial. The theoretical framework is validated through comprehensive mathematical analysis and simulations, complemented by a real-world case study using data from an actual installation. Results demonstrate the method's effectiveness in handling non-sinusoidal conditions through both theoretical cases and actual power system measurements. Furthermore, a parametric analysis of the real-world data reveals a key practical insight: a reduced-order compensator, targeting only the most dominant harmonics, can achieve nearly all of the source current reduction provided by a full compensator, thus offering an optimal trade-off between cost and performance. This research contributes to power systems theory by providing a computationally efficient and flexible approach for power quality enhancement in modern distribution systems.

1. Introduction

1.1. Motivation

The transition towards sustainable energy systems and smart grids has intensified the need for advanced mathematical methods to optimize power flow and enhance grid efficiency. While modern power networks increasingly integrate renewable energy sources and smart grid technologies, they face growing challenges in power quality management, particularly regarding reactive current compensation under non-sinusoidal conditions. These challenges are especially relevant in active distribution networks and microgrids, where power electronic interfaces and renewable energy sources introduce significant harmonic distortion.

Traditional reactive power compensation methods, while effective in conventional power systems, face substantial limitations in modern smart grid environments. Existing analytical approaches often yield

non-physical solutions with negative reactive elements, making them impractical for implementation in active distribution networks. Furthermore, these methods typically lack the mathematical rigor and flexibility required to handle the complex load configurations characteristic of modern power systems with high renewable energy penetration.

To address these limitations, mathematical programming approaches for reactive current compensation have gained attention. This strategy focuses on compensating the current component in quadrature with the supply voltage, offering a more rigorous mathematical foundation compared to traditional reactive power compensation. For clarity, this study adopts the term “reactive current” to denote the portion of the current that can be compensated through optimization of passive and lossless reactive elements.

While active power electronic compensators offer superior dynamic performance, their high cost and complexity often make them unsuitable for widespread deployment in distribution networks. In contrast,

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optimally designed passive compensators provide a cost-effective solution for reactive current compensation in smart grid applications, despite some limitations compared to active solutions.

The growing complexity of modern electrical networks, particularly with the integration of renewable energy sources, demands novel optimization approaches for power quality enhancement [1,2]. This paper presents a mathematical programming method that overcomes existing limitations, offering a theoretical framework for optimizing power flow in modern distribution networks while supporting the transition towards more sustainable and efficient electrical systems.

1.2. Background and literature review

The analysis of power properties in electrical circuits with non-sinusoidal waveforms has been a subject of scientific scrutiny for more than a century. In the seminal work of Budeanu in 1927, reactive and distortion power were defined for circuits with non-sinusoidal voltages and currents [3]. However, these definitions encountered criticism over the following decades due to their lack of clear physical interpretation and questionable practical utility. This led to the development of alternative frameworks based on time-domain analysis and different power decompositions, proposed by Fryze [4], Enslin & van Wyk [5], and Depenbrock with his FBD-method [6], among others.

A particularly influential framework is the Currents' Physical Components (CPC) theory developed by Czarnecki, which decomposes the current into components with claimed physical meaning (active, scattered, and reactive) [7,8]. This theory provides a foundation for identifying the precise current that must be compensated using reactive elements to optimize power delivery, which corresponds to the *reactive current* this paper aims to eliminate. The focus on current compensation is widely regarded as more aligned with physical reality than abstract power concepts, especially under non-sinusoidal conditions [9].

The practical goal of this compensation is to improve power quality, a field where both active and passive solutions are employed. Active Power Filters (APFs) offer superior performance and flexibility but at a significantly higher cost [10]. Consequently, the design of cost-effective passive *LC* compensators remains a highly relevant area of research. The synthesis of these passive compensators has been approached from two main perspectives.

The first approach relies on numerical optimization. These methods define an objective function—such as minimizing transmission losses or maximizing the power factor—and use iterative algorithms to find the optimal component values. Works like those by Aziz et al. [11] and Maza Ortega et al. [12] exemplify this strategy, providing powerful tools for filter design, particularly when dealing with complex constraints like resonance phenomena.

A second approach, which this paper follows, seeks a direct analytical or algebraic solution. The aim is to develop a systematic, non-iterative method for calculating the compensator components. The original synthesis methods based on CPC theory were often non-linear and algorithmically complex [13]. A significant recent advancement was made by Jeon (2020), who successfully linearized the problem into a matrix equation, $\mathbf{B} = \mathbf{A}\mathbf{C}$, allowing the unknown capacitances $\mathbf{C}$ to be found via simple matrix inversion [14].

However, this elegant linearization has a critical practical limitation: the matrix inversion can yield negative capacitance values, which are physically unrealizable. Jeon acknowledges this, stating that if a negative value is obtained, “the resonant frequencies are to be rearranged” [14]. This corrective action is described as a non-algorithmic, trial-and-error process. The lack of a defined procedure for this crucial step makes the method incomplete from an algorithmic standpoint and hinders its reproducibility, as there is no systematic way to guarantee a valid solution. Other recent works, such as Pană et al. [15], also tackle the design of passive compensators aiming for only capacitive solutions, but their approach is based on load sequence currents rather than a direct solution for harmonic susceptances. This leaves a clear gap for a

method that is both linear-algebraic and robustly guarantees a physically implementable outcome by construction.

1.3. Contributions

This paper introduces a new algorithm for computing optimal passive and lossless *LC* elements in single-phase circuits under non-sinusoidal conditions. The algorithm offers a novel method for compensating reactive current that overcomes the limitations of traditional approaches. The method proposed in this work can adapt to a wide range of operational scenarios, including those with sub- or inter-harmonics, without requiring negative capacitance or inductance values. This is a common challenge in traditional linear equation systems. The proposed method simplifies the computational process by using linear programming principles to achieve total reactive current compensation. This approach enhances both the theoretical and practical understanding of reactive and harmonic current compensation while streamlining implementation.

1.4. Outline

The remainder of the paper is organised as follows: Section 2 reviews necessary background on non-sinusoidal systems and limitations of existing compensation methods, highlighting the need for an improved reactive current compensation approach. Section 3 details the proposed compensation technique, including the compensator topology selection, positioning of resonant frequencies, derivation of the compensation equation set, and application of linear programming to avoid non-physical solutions. Section 4 demonstrates the efficacy of the method through simulations on systems with limited and extensive harmonics. It also validates performance under real-world conditions using experimental measurements, followed by Section 5, which concludes the paper with a summary of findings and potential avenues for future research.

2. Theoretical framework and methodology

The proposed technique for compensating reactive current is based on a solid theoretical foundation, which includes a precise algorithm for computing passive and lossless *LC* elements that are critical to the compensation process. This method provides an innovative approach to traditional reactive power methods, offering improved adaptability to a variety of operating conditions without the need for negative reactive components, which are often a constraint in existing methods.

2.1. Current compensation under non-sinusoidal voltage supply

Current compensation in sinusoidal and steady-state circuits are widely known and very well understood. However, non-sinusoidal supplies still remain as a challenge that can potentially be addressed in either the time or frequency domain. Every approach has its benefits and drawbacks. An advantage of using the frequency domain is that passive elements can be used to partially compensate for very specific current components, which can significantly reduce the cost of the compensator by eliminating the need for a fully power electronics-based compensator.

Let us examine the general single-phase 1-port passive compensator in Fig. 1. The internal structure can be as simple as a single element (either a capacitor *C* or an inductor *L*) or it can be more complex, incorporating *LC* combinations according to Foster or Cauer [16] configurations, with more sophisticated models based on network synthesis theory [17].

According to the main power theories [18,19], the load current i_l can be decomposed into different terms that have engineering significance. For example, a typical decomposition is

$$i_l = i_a + i_r + i_d$$

where the term i_a is known as the active current and is responsible for the active power delivery to the load. The term i_r denotes the reactive current (because of the presence of reactors) and the remainder is the

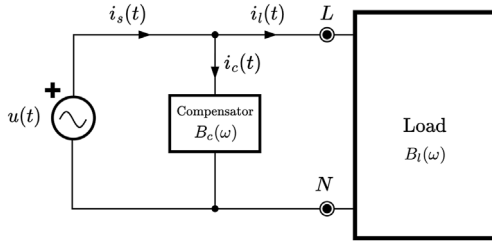
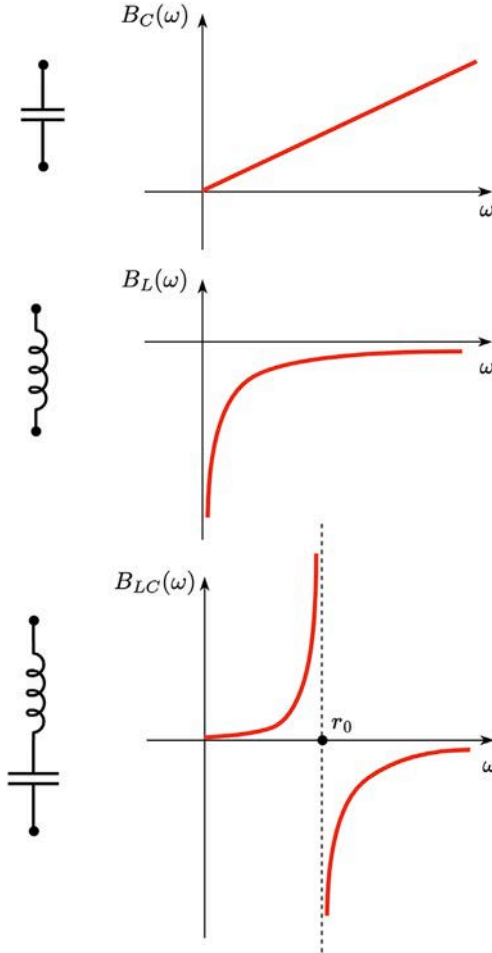


Fig. 1. Setup of a single-phase load and 1-port general compensator.

Fig. 2. Susceptance's frequency response for some typical compensators. Top) Capacitor $B_C = C\omega$, middle) Inductor $B_L = -1/L\omega$ and bottom) LC series $B_{LC} = C\omega/(1 - LC\omega^2)$.

distorted current i_d . Neither of these two terms contributes to active power in the load, so their removal is sought. However, only i_r can be removed using a specific combination of lossless passive elements like L or C . This process is equivalent to identifying the suitable susceptance values $B_c(\omega)$ in a compensator that oppose those of the load $B_l(\omega)$ at every considered frequency ω . In this new scenario, the current provided by the source is

$$i_s = i_l + i_c = i_a + i_d$$

where $i_c = -i_r$. Consequently, it is crucial to determine the suitable arrangement of elements that fulfills this requirement. The frequency responses of the susceptance $B(\omega)$ for typical layouts of compensators are illustrated in Fig. 2. By selecting an appropriate combination of these elements, the intended purpose can be realised.

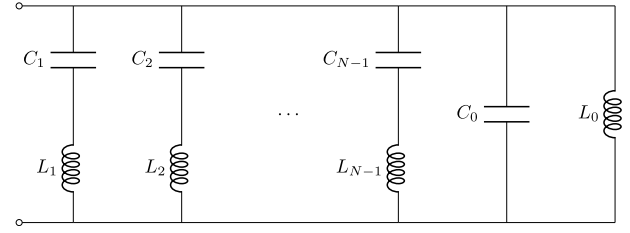


Fig. 3. Proposed circuit compensator based on Foster-II topology.

Therefore, to achieve full reactive current compensation at N distinct frequencies, a passive compensator must possess sufficient degrees of freedom to independently match the target susceptance at each frequency. The number of degrees of freedom in a lossless one-port network is determined by the number of its poles and zeros, which in turn depends on the number of reactive elements, M . Foundational work in network synthesis theory, discussed by Czarnecki in the context of harmonic compensation [13], establishes that the number of elements required is bounded by $N \leq M \leq 2N-1$. The upper bound, $M = 2N-1$, represents the complexity required for the most general case to guarantee a solution. Our proposed method starts from a topology that satisfies this condition, and then uses an optimization procedure to simplify it.

3. Proposed method

This section presents the proposed reactive current compensation method. The method guarantees full reactive compensation under any reasonable number of frequencies, arbitrary load susceptance configurations, and without requiring negative capacitor or inductor values. The key aspects of the method are detailed in the following sections.

3.1. Compensator topology

A Foster-II compensator topology is **selected** for this work, consisting of $N-1$ series LC branches in parallel. This is complemented with an additional shunt inductor L_0 and capacitor C_0 in parallel (see Fig. 3). This parallel topology is chosen for its superior robustness from a practical point of view; unlike series-connected configurations (e.g., Foster-I or Cauer-I forms), the failure of a single LC branch does not lead to a total disconnection of the compensator but rather a partial degradation of its performance. The initial maximum number of elements in this configuration is $2N$.

The complex admittance is as follows

$$Y(s) = C_0 s + \frac{1}{L_0 s} + \sum_{k=1}^{N-1} C_k \frac{r_k^2 s}{s^2 + r_k^2} \quad (1)$$

where $r_k = \frac{1}{\sqrt{L_k C_k}}$ is the resonant frequency and s is the complex frequency.

This topology is uniform because the number of LC branches only depends on the number of frequency components N . It is also adaptable because the configuration can be reduced by eliminating non-critical branches, as identified in subsequent steps. This reduction decreases cost without significantly impacting compensation efficacy.

3.2. Resonant frequency selection

The resonant frequencies r_k for each LC branch k must be defined a priori and positioned between each adjacent pair of supply frequencies to comply with Foster theorem. Setting $N-1$ resonant frequencies ensures the compensation equations yield a well-defined and invertible matrix for the linearised system of equations. Any continuous and monotonic function can determine suitable resonant frequencies, but the arithmetic mean is selected here for simplicity

$$r_k = \frac{\omega_k + \omega_{k+1}}{2} \quad (2)$$

It is crucial that the resonant frequencies remain between adjacent supply frequencies for the method to guarantee full compensation. The precise value of r_k impacts the required element values but not the compensation efficacy. Further optimization of resonant frequencies based on cost, stability margins or other criteria may be studied in the future.

3.3. Compensation equations

The admittance of a passive lossless compensator is purely imaginary, so the complex frequency in (1) is evaluated at $s = j\omega$, leading to

$$Y(\omega) = \text{Im}\{Y(j\omega)\} = C_0\omega - \frac{1}{L_0\omega} + \sum_{k=1}^{N-1} C_k \frac{r_k^2 \omega}{r_k^2 - \omega^2} \quad (3)$$

For mathematical convenience, (3) is divided by ω :

$$y(\omega) = \frac{Y(\omega)}{\omega} = C_0 - \frac{1}{L_0\omega^2} + \sum_{k=1}^{N-1} C_k \frac{r_k^2}{r_k^2 - \omega^2} \quad (4)$$

This equation must hold for each of the N harmonic frequencies, ω_n , present in the system. To achieve full compensation, the compensator's susceptance must be the negative of the load's susceptance, B_n , at each frequency. Therefore, the target value for $y(\omega_n)$ is:

$$y_n = -\frac{B_n}{\omega_n} \quad (5)$$

By enforcing this condition, the total susceptance at the point of common coupling becomes zero for each harmonic frequency. This achieves the method's objective of full reactive current compensation, corresponding to a 100 % reactive compensation index by design.

Evaluating (4) for all N frequencies results in a linear system of equations, which can be expressed in matrix form as:

$$\mathbf{y} = \mathbf{A}_\star \mathbf{b} \quad (6)$$

where the vectors and the matrix are defined as:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} C_0 \\ 1/L_0 \\ C_1 \\ \vdots \\ C_{N-1} \end{bmatrix},$$

$$\mathbf{A}_\star = \begin{bmatrix} 1 & -\frac{1}{\omega_1^2} & \frac{r_1^2}{r_1^2 - \omega_1^2} & \cdots & \frac{r_{N-1}^2}{r_{N-1}^2 - \omega_1^2} \\ 1 & -\frac{1}{\omega_2^2} & \frac{r_1^2}{r_1^2 - \omega_2^2} & \cdots & \frac{r_{N-1}^2}{r_{N-1}^2 - \omega_2^2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & -\frac{1}{\omega_N^2} & \frac{r_1^2}{r_1^2 - \omega_N^2} & \cdots & \frac{r_{N-1}^2}{r_{N-1}^2 - \omega_N^2} \end{bmatrix}$$

The system has N equations but $N+1$ unknowns, making it underdetermined. To find a unique solution, we treat the shunt capacitance C_0 not as a direct unknown, but as a control parameter to be optimized later (see Section 3.4). This is achieved by algebraically separating the first column of $\mathbf{A}_\star$, which is associated with C_0 :

$$\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} C_0 + \begin{bmatrix} -\frac{1}{\omega_1^2} & \cdots & \frac{r_{N-1}^2}{r_{N-1}^2 - \omega_1^2} \\ \vdots & \ddots & \vdots \\ -\frac{1}{\omega_N^2} & \cdots & \frac{r_{N-1}^2}{r_{N-1}^2 - \omega_N^2} \end{bmatrix} \begin{bmatrix} 1/L_0 \\ C_1 \\ \vdots \\ C_{N-1} \end{bmatrix} \quad (7)$$

Rearranging this equation yields a solvable $N \times N$ system:

$$\mathbf{A}\mathbf{x} = \mathbf{y} - \mathbf{c}_0 \quad (8)$$

where the new $N \times N$ matrix $\mathbf{A}$ and the new $N \times 1$ unknown vector $\mathbf{x}$ are:

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{\omega_1^2} & \frac{r_1^2}{r_1^2 - \omega_1^2} & \cdots & \frac{r_{N-1}^2}{r_{N-1}^2 - \omega_1^2} \\ -\frac{1}{\omega_2^2} & \frac{r_1^2}{r_1^2 - \omega_2^2} & \cdots & \frac{r_{N-1}^2}{r_{N-1}^2 - \omega_2^2} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{\omega_N^2} & \frac{r_1^2}{r_1^2 - \omega_N^2} & \cdots & \frac{r_{N-1}^2}{r_{N-1}^2 - \omega_N^2} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 1/L_0 \\ C_1 \\ \vdots \\ C_{N-1} \end{bmatrix}$$

and $\mathbf{c}_0$ is a column vector of size N with all entries equal to C_0 . The solution for the remaining compensator parameters is then found by matrix inversion:

$$\mathbf{x} = \mathbf{A}^{-1}(\mathbf{y} - \mathbf{c}_0) = \mathbf{M}(\mathbf{y} - \mathbf{c}_0) \quad (9)$$

where $\mathbf{M} = \mathbf{A}^{-1}$. Practical implementations should solve the system using efficient methods (LU decomposition, QR factorization) rather than explicit matrix inversion, as discussed in detail in Section 3.7. Note that the value of C_0 is still undetermined at this stage and will be found using the optimization procedure described in the next section.

3.4. Managing negative solutions with linear programming and simplex method

Linear compensation equations can result in non-physical negative solutions for passive capacitance or inductance. To preclude this, principles from linear programming and the simplex method are applied. The compensation condition in (9) can be expressed in vector form as

$$\mathbf{u}_n \cdot \mathbf{x} = z_n \quad (10)$$

where $\mathbf{u}_n$ is the n -th row vector in matrix $\mathbf{A}$ and $z_n = y_n - C_0$. Vector $\mathbf{x}$ can then be decomposed into parallel and perpendicular components to $\mathbf{u}_n$:

$$\mathbf{x} = \mathbf{x}_\parallel + \mathbf{x}_\perp = \frac{z_n}{u_n^2} \mathbf{u}_n + \mathbf{x}_{n\perp} \quad (11)$$

The orthogonal vectors $\mathbf{x}_{n\perp}$ form the hyperplane representing the simplex face that nullifies each variable x_k . By systematically adjusting C_0 , the optimal value can be found that avoids negative solutions before the first $x_k = 0$. By setting each $x_k = 0$, we have

$$\sum_n M_{kn}(y_n - C_{0k}) = 0 \quad (12)$$

and

$$C_{0k} = \frac{\sum_n M_{kn} y_n}{\sum_n M_{kn}} \quad (13)$$

The variable C_{0k} represents the capacitance value for which branch k gets fully compensated, making it redundant. The maximum non-negative C_{0k} over all branches gives the simplest configuration that eliminates at least one branch while keeping all solutions positive. This value gives an optimized C_0 , achieving the compensator simplification.

Note that the simplex method has the additional advantage of allowing a branch to be eliminated - specifically, the branch whose capacitance is nullified in the previous step. This leaves an N -branch $(2N-1)$ -compensator that still guarantees full compensation of the N frequency components.

3.5. Further branch reduction

If any additional C_{0k} values are only slightly below the final C_0 , the corresponding branches can also be neglected since their contribution is minor. A threshold parameter d determines the minimum $C_0 - C_{0k}$ gap to eliminate extra branches. Parameter d depends on the acceptable reduction in compensation efficacy.

3.6. Regulating resonant frequencies

By adjusting the resonant frequencies r_k , it may be possible to reach a simplex edge that nullifies a second branch while retaining total compensation. Further manipulation of r_k can cumulatively eliminate additional branches in some cases.

The potential for further optimization via adjusting r_k requires additional investigation. Alternative cost, stability and performance metrics could guide the resonance frequency selection.

3.7. Algorithm properties and practical considerations

Beyond its primary function, the proposed algorithm has several inherent properties relevant to its practical application in terms of both its computational characteristics and its economic implications.

3.7.1. Automatability, robustness, and complexity

The procedure detailed in Algorithm 1 addresses several key practical concerns through its fundamental design characteristics. Its deterministic and non-iterative nature, based on a direct sequence of algebraic operations, makes it fully suitable for automated implementation without requiring manual tuning or convergence monitoring.

The algorithm's computational complexity is dominated by solving the linear system $\mathbf{Ax} = \mathbf{y} - \mathbf{c}_0$, which has a complexity of $O(N^3)$ for direct matrix inversion, where N is the number of compensated harmonics. However, practical implementations should employ specialized linear system solvers such as LU decomposition, QR factorization, or Cholesky decomposition, which reduce the effective complexity to $O(N^2)$ after an initial factorization step while providing superior numerical stability. Modern numerical libraries such as LAPACK, Eigen (C++), or NumPy/SciPy (Python) implement these algorithms with highly optimized routines that exploit matrix structure. For typical power quality applications where $N < 50$ harmonics are considered, computational time on modern platforms should be on the order of milliseconds, making real-time implementation practical even on embedded industrial controllers.

Moreover, the algorithm's analytical foundation in a well-conditioned linear system ensures numerical stability. Small perturbations in input measurement data (e.g., harmonic amplitude or phase uncertainties) result in correspondingly small and bounded changes to the calculated component values, providing predictable behavior under realistic measurement conditions.

Regarding the computational advantages over alternative approaches, the key advantage lies in the problem formulation itself. By linearizing what would otherwise be a nonlinear constrained optimization problem, the method guarantees convergence in a single deterministic pass. In contrast, general-purpose nonlinear optimization approaches

(such as sequential quadratic programming or interior point methods) require iterative refinement with uncertain convergence properties, particularly when handling the non-convex constraint space imposed by positive reactive element requirements. Furthermore, our method inherently ensures physically realizable solutions without post-processing verification or penalty functions, eliminating computational overhead present in alternative formulations. Direct computational comparison with previous analytical methods from the literature (such as those by Czarnecki and Jeon) is not feasible due to incomplete algorithmic descriptions in those works and differences in problem formulation, but our single-pass linearized approach offers clear structural advantages over iterative nonlinear alternatives.

3.7.2. Cost considerations

While a detailed economic analysis is beyond the scope of this paper, the proposed algorithm contributes to a cost-effective design in two ways. First, by providing a direct, non-iterative solution, it minimizes engineering time compared to trial-and-error approaches. Second, and more importantly, the linear programming step that determines the optimal C_0 naturally simplifies the compensator. By setting the capacitance of at least one branch to zero (see Section 3.4), the algorithm inherently designs the simplest possible topology that can achieve full compensation, directly reducing the number of physical inductors and capacitors required and thus lowering the overall implementation cost.

3.8. Targeting specific power factor levels

For practical applications requiring a specific power factor target rather than complete compensation, two strategies can be employed within the framework of the proposed method:

- **Sequential branch elimination approach:** After computing the complete solution with all $N - 1$ series LC branches, some of them can be systematically removed based on their contribution to power factor improvement. This can be achieved by computing the power factor for progressively reduced configurations and eliminating branches with minimal impact until the target PF threshold is reached. This approach guarantees that the target is achievable and provides insight into the trade-off between system complexity and compensation effectiveness.
- **A priori branch reduction:** By analyzing the harmonic spectrum and load susceptance pattern, dominant frequency components can be identified before solving the compensation problem. Only branches corresponding to harmonics with significant reactive current contributions would be included in the initial formulation, resulting in a reduced-order system that targets major power quality degradation sources. This approach reduces computational requirements but requires careful harmonic analysis.

The trade-off between compensation effectiveness and system complexity (number of branches, total reactive element ratings, installation cost) represents an interesting avenue for future multi-objective optimization research. Such approaches could directly incorporate power factor targets, cost constraints, and practical implementation limits into the optimization framework, potentially using techniques such as genetic algorithms, particle swarm optimization, or mixed-integer linear programming.

4. Illustrative examples

To demonstrate the effectiveness and practicality of the proposed reactive current compensation method, we present three illustrative examples. These examples encompass simulated scenarios with varying levels of harmonic distortion and a real-world case study from an electrical installation. The chosen scenarios reflect typical challenges encountered in electrical engineering practice, such as the presence of multiple harmonics and significant current distortion. The first example considers a non-sinusoidal voltage source supplying a linear RL load

Algorithm 1 Compensation Method.

- 1: **Define** set of frequencies N and load susceptances
 - 2: **Compute** $y_n = -\frac{b_n}{\omega_n}$ terms
 - 3: **Select** resonant frequencies r_k
 - 4: **Calculate** load terms y_n and matrix terms M_{nk}
 - 5: **Determine** optimal capacitor value C_0
 - 6: **Solve** the system of equations
 - 7: **Determine** branches to be eliminated
 - 8: **Compute** L_k values from C_k and r_k
 - 9: **Output** element values
-

with a limited number of harmonics. The second scenario is more complex, with 17 harmonic frequencies in the supply. Finally, we discuss a third example involving measurements from a real-world facility.

4.1. Non-sinusoidal supply with a reduced number of harmonics

Consider a circuit where a series RL load, with $R = 15\Omega$ and $L = 0.0796$ H, is supplied by a non-sinusoidal voltage source. The source contains three harmonic components, corresponding to the fundamental, third, and fifth harmonics. The fundamental angular frequency is $\omega_0 = 100\pi$ rad/s (50 Hz). The set of $N = 3$ angular frequencies present in the supply is therefore:

$$\omega_1 = 1 \cdot \omega_0 = 100\pi \text{ rad/s}$$

$$\omega_2 = 3 \cdot \omega_0 = 300\pi \text{ rad/s}$$

$$\omega_3 = 5 \cdot \omega_0 = 500\pi \text{ rad/s}$$

The voltage as a function of time, $v(t)$, is explicitly defined by these frequencies as:

$$v(t) = 360\sqrt{2}\sin(\omega_1 t) + 144\sqrt{2}\sin(\omega_2 t) + 42\sqrt{2}\sin(\omega_3 t)$$

Since there are $N = 3$ harmonics to be compensated, the compensator will consist of $N - 1 = 2$ series LC branches, in parallel with a shunt inductor L_0 and a shunt capacitor C_0 .

We shall initially establish the resonant frequencies of every branch using (2), so the values are

$$r_1 = \frac{\omega_1 + \omega_2}{2} = 2\omega_0 = 200\pi \quad (14)$$

$$r_2 = \frac{\omega_2 + \omega_3}{2} = 4\omega_0 = 400\pi \quad (15)$$

with $\omega_n = n\omega_0$. The matrix A can now be computed as

$$A = \begin{bmatrix} -10.13 \times 10^{-6} & 1.333 & 1.066 \\ -11.26 \times 10^{-7} & -0.800 & 2.286 \\ -40.52 \times 10^{-8} & -0.191 & -1.777 \end{bmatrix} \quad (16)$$

The absolute value of the susceptance for a series RL load is computed as

$$B_n = -\frac{L\omega_n}{R^2 + (L\omega_n)^2} \quad (17)$$

resulting in the following values for the three harmonics

$$B_1 = -0.02940 \text{ S}$$

$$B_2 = -0.01282 \text{ S}$$

$$B_3 = -0.00789 \text{ S}$$

Recalling (5), the independent vector yields

$$y_1 = 9.3608 \times 10^{-5}$$

$$y_2 = 1.3599 \times 10^{-5}$$

$$y_3 = 0.50198 \times 10^{-5}$$

Using the linear programming method, the required parallel capacitor is $C_0 = 28.128 \times 10^{-6}$ F. Finally, solving the matrix system, the capacitors in the series LC branches and the parallel inductor L_0 are obtained

$$C_1 = 42.34 \times 10^{-6} \text{ F}$$

$$C_2 = 8.42 \times 10^{-6} \text{ F}$$

$$1/L_0 = 0 \implies L_0 \rightarrow \infty$$

The algorithm yields a value for $1/L_0$ that is zero (or numerically very close to zero), which implies that the required parallel inductance

Table 1

Performance metrics for Case A.

Metric	Before compensation	After compensation	Difference (%)
Current (RMS)	12.49	6.36	-49.08
Power Factor (PF)	0.48	0.94	95.83
Current THD (%)	15.48	6.18	-60.08

Table 2

Harmonic voltages and susceptances for Case B.

Frequency (Hz)	Amplitude (V)	Phase (rad)	Conductance (S)	Susceptance (S)
50	230.0	0.00	0.010	-0.050
150	12.5	-1.10	0.002	-0.095
250	14.1	0.80	0.005	0.082
350	8.9	-0.50	0.008	-0.075
420	3.2	1.50	0.002	0.061
550	7.1	-2.10	0.0002	-0.055
650	5.5	0.30	0.00025	0.048
810	2.1	-0.90	0.00028	-0.041
1150	3.8	1.80	0.00030	0.035
1230	1.9	-2.50	0.00032	-0.029

L_0 approaches infinity. From a physical standpoint, an infinite inductance has zero admittance ($Y_{L_0} = 1/(j\omega L_0) \rightarrow 0$) and acts as an open circuit. Therefore, it can be neglected from the final compensator design with negligible impact on the compensation performance, while simplifying the implementation and reducing costs.

The remaining inductors in the series LC branches are

$$L_1 = \frac{1}{C_1 r_1^2} = \frac{1}{(42.34 \times 10^{-6})(200\pi)^2} = 59.82 \text{ mH}$$

$$L_2 = \frac{1}{C_2 r_2^2} = \frac{1}{(8.42 \times 10^{-6})(400\pi)^2} = 74.83 \text{ mH}$$

It can be verified that this arrangement gives zero imaginary impedance at each ω_n , so the supply current from the source is minimal. In this scenario, the load requires 12.49 A. Compensation reduces this to 6.36 A, a reduction of approximately 49 %. The performance improvement in terms of standard power quality metrics is summarized in Table 1. As a result of the current reduction, transmission and distribution losses are reduced by approximately 76 %.

4.2. Non-sinusoidal supply with a high number of harmonics

A more complex example is presented here, in which a non-sinusoidal voltage source comprising 10 significant harmonics is applied to a generic load with a poor initial power factor. This case serves to demonstrate the algorithm's effectiveness and scalability in a scenario with rich harmonic content (Table 2).

The proposed method was applied to determine the optimal compensator parameters. The resulting values for the main parallel capacitor (C_0) and inductor (L_0), as well as for the nine series LC branches, are detailed in Table 3. Of particular interest is the result for the ninth branch,

Table 3

Compensator component values for Case B.

# Branch	Capacitance C (μ F)	Inductance L (mH)
0	27.5850	43.0510
1	173.4252	14.6059
2	77.4857	8.1726
3	15.1416	18.5878
4	11.3958	14.9959
5	8.8819	12.1241
6	5.9975	11.7318
7	3.6184	13.1366
8	7.1996	3.6634
9	0.0000	∞

Table 4

Performance metrics for Case B.

Metric	Before Compensation	After Compensation	Difference (%)
Current (A RMS)	11.88	2.30	−80.62
Power Factor (PF)	0.193	0.998	416.34
Current THD (%)	15.93	4.50	−71.76

where the algorithm yielded a capacitance of zero ($C_9 = 0$). This is a key feature of the proposed method: it automatically identifies and eliminates non-essential branches by driving their capacitance to zero, thus simplifying the final hardware implementation without sacrificing compensation performance.

A noticeable improvement in power quality is quantified in Table 4. The compensation reduces the RMS current drawn from the source by over 80 % and improves the power factor from a very low 0.193 to near unity. Furthermore, the total harmonic distortion of the current is reduced by more than 71 %. These results confirm that the proposed synthesis method is highly efficient and reliable, providing physically realizable component values that lead to a significant enhancement of the system's performance. The efficiency, simplicity, and reliability of the method make it an attractive option for designing optimal lossless branch compensators.

4.3. Real-world case

This final example presents experimental measurements acquired from a high-power horticultural packaging and handling facility with an installed capacity of 200 kW. The industrial environment exhibits significant harmonic distortion due to the extensive machinery present in the installation but also an important phase delay is visible ($\varphi \approx 40^\circ$). Voltage and current waveforms were captured using an openmeter device [20] through a dedicated application programming interface at 50 kHz sampling rate. As evidenced in Fig. 4, substantial distortion in the current waveform was observed. Fourier analysis depicted in Fig. 5 further reveals numerous high-amplitude frequency components in the current spectrum, and accordingly, Fig. 6 shows the measured susceptance pattern across the harmonic range.

Based on the obtained data, our proposed compensation method produced a custom reactive compensator to address the identified harmonics. According to our initial calculations presented in Table 5, complete cancellation of reactive current is theoretically achievable as shown in Fig. 7. While the solution yields 49 LC branches (with one being null by default), it is important to emphasize that this comprehensive solution serves primarily as a theoretical demonstration of the method's capabilities rather than a practical implementation proposal. The high number of compensator branches, although mathematically

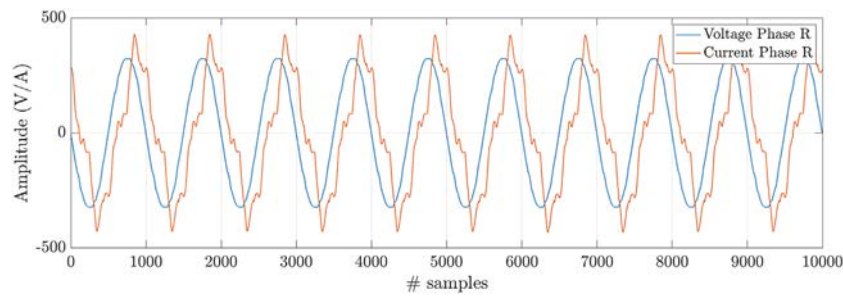


Fig. 4. Voltage and current waveforms in Case C. Note the phase delay and high distortion of the current.

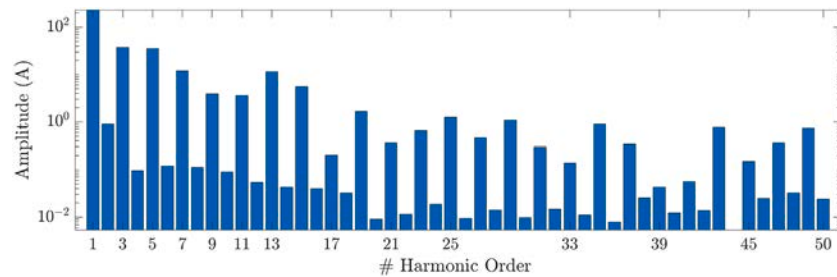


Fig. 5. The spectrum of the current for the load of Case C.

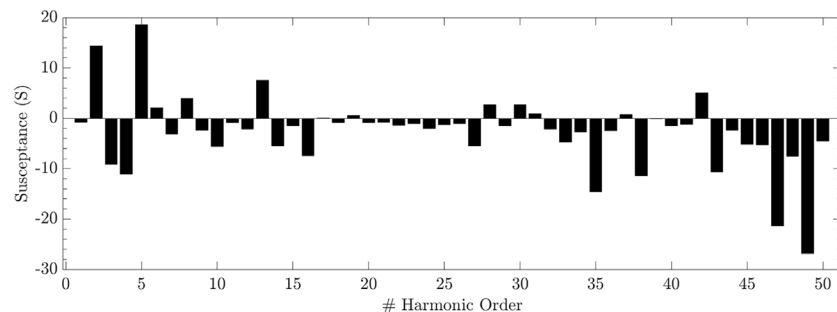


Fig. 6. Susceptances of the first 50 harmonics for the load in Case C.

Table 5
Series LC branch parameters for Case C.

# LC	L (mH)	C (μ F)	# LC	L (mH)	C (μ F)
0	0.083	3268.300	25	0.130	119.590
1	0.102	44,273.000	26	0.139	103.810
2	0.169	9607.200	27	0.118	113.690
3	0.111	7427.900	28	0.143	87.302
4	0.077	6460.300	29	0.134	87.106
5	0.137	2444.500	30	0.156	69.912
6	0.127	1890.900	31	0.172	59.532
7	0.107	1677.900	32	0.176	54.515
8	0.128	1097.900	33	0.167	53.986
9	0.121	924.490	34	0.213	39.917
10	0.106	863.630	35	0.133	60.641
11	0.112	687.000	36	0.155	48.999
12	0.102	635.660	37	0.224	32.184
13	0.142	390.960	38	0.145	47.146
14	0.115	418.600	39	0.186	34.869
15	0.125	336.270	40	0.199	30.966
16	0.104	356.980	41	0.206	28.567
17	0.115	288.540	42	0.521	10.762
18	0.115	258.360	43	0.243	22.003
19	0.122	219.060	44	0.380	13.463
20	0.122	198.390	45	0.474	10.329
21	0.124	176.270	46	5.461	0.858
22	0.124	160.790	47	0.308	14.558
23	0.128	142.840	48	—	0.000
24	0.127	133.140	49	0.239	17.291

Table 6
Performance metrics for the real-world Case C.

Metric	Before compensation	After compensation	Difference (%)
Current (A RMS)	237.17	152.32	−35.78
Power Factor (PF)	0.63	0.98	55.55
Current THD (%)	23.63	20.43	−13.54

optimal, would likely be impractical in real-world installations due to cost, space, and maintenance considerations.

The simulated implementation of our compensation method demonstrates significant power quality improvement under these challenging industrial conditions. The key performance indicators are summarized in Table 6. The results show a substantial reduction in the RMS current drawn from the source and a marked improvement in both the power factor and the current's total harmonic distortion. Future research will explore the trade-off between the number of compensator branches and power quality enhancement to identify practical implementations.

4.3.1. Performance analysis of reduced-order compensators

While the full 49-branch design provides a theoretical benchmark for maximum performance, a practical implementation requires a trade-off between cost and effectiveness. The primary objective of compensation is to minimize the RMS current drawn from the source, thereby reducing transmission losses. To rigorously evaluate how the

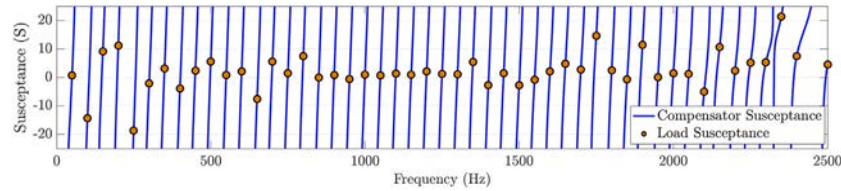


Fig. 7. Susceptance function (blue) for the compensator in Case C. Red points are the particular susceptance values of the load at specified frequencies. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

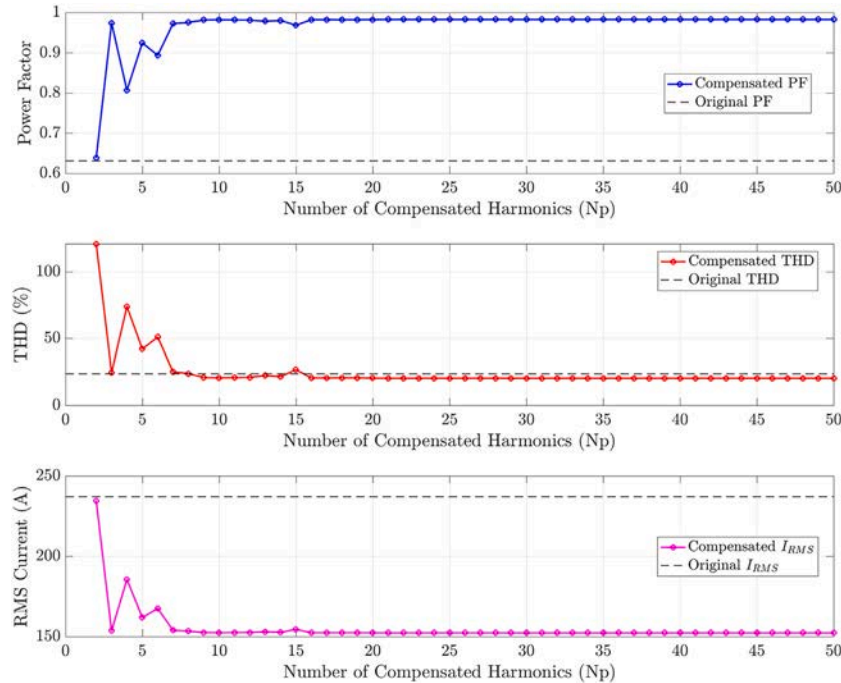


Fig. 8. Evolution of key power quality metrics as the number of compensated harmonics (N_p) increases. The plots demonstrate a clear pattern of diminishing returns, where the most significant improvements are achieved with a small number of targeted harmonics.

Table 7

Performance comparison for uncompensated, full, and reduced-order compensators.

Metric	Uncompensated	Reduced ($N_p = 7$)	Reduced ($N_p = 10$)	Full ($N_p = 50$)
Current (RMS) [A]	237.17	153.90	152.43	152.32
Power Factor (PF)	0.631	0.973	0.982	0.983
Current THD [%]	23.63	25.19	20.80	20.43

number of compensated harmonics impacts this objective, we conducted a parametric analysis. We systematically varied the number of compensated harmonics, N_p , from 2 to 50. For each value of N_p , we selected the harmonics with the highest current amplitudes as compensation targets and synthesized the corresponding optimal compensator using our algorithm.

The evolution of key power quality metrics as a function of N_p is presented in Fig. 8. The plots clearly illustrate the law of diminishing returns. The RMS current of the source decreases dramatically as the first few dominant harmonics are compensated, rapidly approaching the minimum value. Beyond approximately $N_p = 10$, adding more LC branches yields progressively smaller, almost negligible, gains in current reduction. Notably, the power factor exhibits non-monotonic behavior at low N_p values, briefly dipping before improving. This occurs because a passive LC compensator, while designed for N_p specific target frequencies, presents a continuous frequency response across the entire spectrum. At non-target harmonics, the compensator's uncontrolled admittance can inject reactive current with unfavorable phase relationships, temporarily degrading power quality. This interference effect diminishes as N_p increases and more harmonics are explicitly compensated. The full compensator design eliminates this issue entirely by optimizing for all harmonics present.

Table 7 quantifies this trade-off for specific, practical design choices. The performance of reduced-order compensators targeting the top 7 and 10 harmonics is compared against the uncompensated system and the theoretical ideal (full compensation).

The results are conclusive. The maximum possible RMS current reduction (from 237.17 A to 152.32 A) is 84.85 A. A 7-branch compensator, targeting only the most dominant harmonics, reduces the current to 153.90 A, achieving a reduction of 83.27 A. This means that a practical 7-branch design already captures 98.1 % of the maximum achievable benefit. Increasing the complexity to a 10-branch design yields a current of 152.43 A, capturing an impressive 99.9 % of the total possible reduction.

This analysis validates that our algorithm is not a rigid tool for designing overly complex systems, but rather a precise synthesis engine that can be flexibly applied within a practical engineering workflow. It allows designers to identify the optimal point on the cost-benefit curve and implement a highly effective compensator with a minimal number of components.

5. Conclusions

This paper introduces a novel reactive current compensation method for non-sinusoidal power systems using lossless LC networks, offering distinct advantages over existing approaches. It ensures full reactive current compensation for numerous harmonics, employing a systematic LC element value determination process. The method's adaptability to various load susceptance configurations at different harmonic frequencies allows broad practical application. Its incorporation of linear programming ensures positive, realizable element values, addressing the issue of non-physical solutions common in analytical methods. The compensator's design simplifies by eliminating non-essential reactive branches, cutting costs without compromising effectiveness.

Several simulations and a real-world case study validated the method's performance, demonstrating reliable compensation under

diverse conditions and confirming its practicality. Crucially, a parametric analysis on the real-world data revealed that a reduced-order compensator targeting only the most dominant harmonics achieves the vast majority of the total source current reduction, illustrating a clear path for cost-effective practical implementations. This technique represents a notable improvement in reactive current management by focusing on current rather than reactive/apparent power, aligning better with the realities of modern power systems. The algorithm simplifies achieving the desired compensation, improving feasibility.

Future research could explore robust design strategies to handle dynamic industrial conditions, such as significant changes in the load's harmonic profile or small variations in the grid frequency. Additionally, multi-objective optimization approaches could be developed to directly target specified power factor levels while minimizing compensator complexity and cost. The algorithm presented here would serve as the core synthesis tool for designing adaptive solutions, like switched filter banks. The focus of this work has been on the development and validation of the algorithmic framework; the physical implementation and experimental testing of a compensator designed with this method represent a crucial next step. Nevertheless, this work presents a versatile, efficient compensation approach, promising enhanced reactive current management in modern power grids.

CRedit authorship contribution statement

Francisco G. Montoya: Writing – original draft, Methodology, Conceptualization. **Jorge Ventura:** Software. **Xabier Prado:** Methodology, Investigation, Data curation. **Jorge Mira:** Supervision, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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